



## Report on Sabbatical Leave 2019-2020

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The proposed schedule of this Sabbatical included visiting three locations. Two of those locations were visited. A second visit to one of these two locations, as well as the planned visit to the third location, were canceled due to the global pandemic, which, unfortunately, disrupted many other anticipated activities and inhibited meetings, in-person contacts and collaborations and resulted in postponing scheduled talks and other preplanned on-site activities.

The places visited were Athens, Greece, and Aveiro, Portugal. A second scheduled trip to Aveiro, as well as a planned visit to Braga, Portugal, did not materialize. Moreover, scheduled talks at both the National Technical University of Athens and the University of Aveiro had to be postponed for future visits.

In the remaining part of the Report, we focus on what was accomplished and we close with a brief summary of the results. For more details the interested reader is referred to the online monograph <http://voutsadakis.com/RESEARCH/caal.html>, part of which was composed in Sault Sainte Marie in the Fall of 2019 and part of which was authored in Athens and Aveiro in the Spring of 2020, during the Sabbatical Leave.

### **1. Authorship of the Online Monograph**

Both the scope and the content of the monograph have been significantly expanded during the Sabbatical to include virtually all core topics of *Categorical Abstract Algebraic Logic* as well as a variety of more specialized directions, but much still remains to be done, both in terms of polishing some of the chapters, as well as adding in the contents of several of the chapters.

## 2. **The Categorical Leibniz Hierarchy**

The Leibniz hierarchy constitutes one of the cornerstones of the edifice of Abstract Algebraic Logic and, in particular, of the categorical side of the field. In the original proposal, three sub-directions were singled out for special study.

- The first entailed an analysis of the various classes which arise from a particular class of the Leibniz hierarchy of sentential logics and a study of the sub-hierarchies spanned in this fashion. This was accomplished for almost all classes in the semantic hierarchy (Chapters 2-6 of the monograph) and was advanced significantly for a variety of classes in the syntactic hierarchy (Chapters 10-13), even though more work is needed (and is ongoing) in both cases.
- The second entailed an analysis of the interconnections between all semantically defined classes, the interconnections between all syntactically defined ones and, finally the relationships between classes across these two hierarchies. This task was also undertaken, and significant pieces of the hierarchy have been clarified and inserted into the various chapters, as particular classes were studied in detail.
- The last topic involved focusing on the lowest parts of the Leibniz hierarchy; more specifically those classes that lie below protoalgebraic and truth-equational logics. The study of these classes is undertaken in Chapters 5-6 and 13-14. However, it was only moderately advanced during the Sabbatical. These Chapters are expanded and rewritten significantly at present (Academic Year 2020-2021) and more work is expected to be accomplished in subsequent years.

## 3. **Categorical Aspects of the Algebraization Based on Implication**

The aspects of algebraization of sentential and of institutional logics using variants of implication (including order), rather than equivalence, is an important topic in Abstract Algebraic Logic, both for historical reasons and because of potential applications in the study of logical systems, as well as in the analysis of metalogical properties and their correspondence with algebraic properties. Elements of the process were studied as facets in three distinct directions.

- Inside the syntactically defined classes of the hierarchy and, in particular, those classes that subsume prealgebraicity (a property slightly more general than protoalgebraicity);
- Inside the classes in the Gentzen hierarchy;
- Inside the classes in the order hierarchy.

Whereas those classes referred to in the first bullet form part of the main hierarchy and are studied in the main body, Chapters 10-12, the latter are perceived as subjects of more specialized topics and are relegated to later Chapters, namely Chapters 25 and 24, respectively.

#### **4. The Equivalence Theorem in Categorical Terms**

This topic was the one that was the least advanced. However, some aspects of it were contributed in the first sections of Chapter 11. It is, admittedly, a topic that requires further investigations and calls for a much more focused and extensive study than we were able to afford in the Sabbatical. This remains an open direction for future study and for potential collaboration with some of the original contributors.

#### **5. Seminars**

The Fall semester of 2019 was spent in Sault Sainte Marie. Traveling took place during the Spring of 2020 and this was when the corona virus outbreak hit. Therefore, the chance for both attending seminars and speaking in seminars to disseminate the work presented here and advance further potential collaborations was limited. In terms of the first, a Ph.D. Defense presentation was attended at the University of Aveiro that fused Logic with considerations in Chemistry and Biology. On the other hand, planned seminar presentations, at Athens, Aveiro and Braga did not materialize. Since the talks to be given had been prepared (and with the vaccines in the process of being widely available to contain the pandemic), the hope is that new visits will be scheduled soon and the seminars will take place during these upcoming trips.

I am looking forward to continuing this study and to advancing the writing of the monograph, as well as further cultivating and strengthening the contacts and the relationships that made this wonderful opportunity possible.

## **Summary and Results of the Sabbatical Leave:**

- A. I have used the Sabbatical to solidify and strengthen relationships with fellow faculty members at the Athens Polytechnic (Professor Chrysafinos) and at the Universities of Aveiro and Braga (Professors Martins, Madeira and Soares).
- B. Several directions of research in Categorical Abstract Algebraic Logic were advanced and publication of several general and particular results was accelerated through authorship of additional Chapters of the online monograph.
- C. Even though I was unable, because of the pandemic, to give scheduled talks detailing my research, I was able to attend a Ph.D. Defense presentation at Aveiro which made an ambitious attempt in applying logical reasoning to the field of synthetic biology. Moreover, I was able, with joy, to meet and discuss some open problems with a newly appointed faculty member at Aveiro, Professor Madeira, with whom I had collaborated, under the coordination of his advisor, Professor Martins, when he had still been a fresh Ph.D. graduate several years ago, during one of my previous visits to Portugal.

Overall, it was a very rewarding Sabbatical experience, despite the difficulties that arose, and I am looking forward to similar and equally rewarding experiences in the future, if and when the opportunity arises. I am very thankful to all colleagues, both at Sault Sainte Marie and the intended and/or visited destinations (Athens, Aveiro and Braga), who made this enriching and rewarding experience possible through their warmth, their caring and their hospitality.